Test 3 Review

November 20, 2018

1. Differentiate the following functions.
   (a) $f(x) = \log_4 (x + \sqrt{\csc x})$
   (b) $g(x) = x^3 \sin^2 (4x)$
   (c) $h(x) = \log_7 \left(\dfrac{\sqrt{x^2+3} \sin^3 x}{\sqrt[3]{9x}} \right)$

2. $f(x) = 2(x^2 + x^3)^2$
   (a) Find the critical numbers.
   (b) Find the intervals of increase and decrease.
   (c) Find the local extrema using the first derivative test and the second derivative test.

3. $f(x) = -3x^3 + 5x + 1$
   (a) Find the intervals of concavity and inflection points.
   (b) Find the absolute maximum and minimum on the interval $[-1, 1]$.

4. Using the table of values below, find $(f \circ g)'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

5. If $h(x) = \sqrt{4 + 3f(x)}$, $f(1) = 7$, $f'(1) = 4$, find $h'(1)$.

6. Sketch a continuous function f that satisfies the following conditions:
   - $f(-3) = 0$, $f(0) = 3$, $f(2) = 1$, $f(3) = 0$
   - $f'(-3)$ does not exist, $f'(0) = f'(2) = 0$
   - $f'(x) < 0$ on $(-\infty, -3)$, $(0, 2)$, $(2, \infty)$
   - $f'(x) > 0$ on $(-3, 0)$
   - $f''(x) < 0$ on $(\infty, -3)$, $(-3, 1)$, $(2, \infty)$
   - $f''(x) > 0$ on $(1, 2)$

7. Given $\mathbf{a} = \langle 3, 1, 2 \rangle$, and $\mathbf{b} = \langle 7, 6, 5 \rangle$:
   (a) Find $\mathbf{b} - \mathbf{a}$ and its magnitude.
   (b) Is $\mathbf{b} - \mathbf{a}$ a unit vector? If not, find a unit vector in the same direction.

8. An airplane is heading due north at 800 km/h and it encounters a wind blowing east at 100 km/h. Find the resultant ground velocity of the plane.