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- Tutorial 7 Q2.

1.a. $f(x) = \log_4\left(x + \sqrt{\csc x}\right)$

$$\frac{1}{\sin x \sin x} \cos x$$

$$(\csc x)' = \left((\sin x)^{-1}\right)' = -1\,(\sin x)^{-2} \cdot \cos x$$
$$= -\csc x \cot x.$$

Liebniz

$$f'(x) = \frac{d\left(\log_4\left(x+\sqrt{\csc x}\right)\right)}{d\left(x+\sqrt{\csc x}\right)} \cdot \frac{d\left(x+\sqrt{\csc x}\right)}{dx}$$

$$= \text{"} \qquad \text{"} \cdot \left(\frac{dx}{dx} + \frac{d\left(\sqrt{\csc x}\right)}{dx}\right)$$

$$= \text{"} \qquad \text{"} \cdot \left(\frac{dx}{dx} + \frac{d\left(\sqrt{\csc x}\right)}{d(\csc x)} \cdot \frac{d(\csc x)}{dx}\right)$$

$$= \frac{1}{\left(x+\sqrt{\csc x}\right)\ln(4)} \cdot \left(1 + \frac{1}{2\sqrt{\csc x}} \cdot (-\csc x \cot x)\right)$$

$$= \frac{1 - \dfrac{\csc x \cot x}{2\sqrt{\csc x}}}{\left(x+\sqrt{\csc x}\right) \cdot \ln(4)}$$

b). $g(x) = \underbrace{x^3}_{a(x)} \underbrace{\sin^2(4x)}_{b(x)}$

$a'(x) = 3x^2$

$b'(x) = 2\sin(4x) \cdot \cos(4x) \cdot 4 = 8\sin(4x)\cos(4x).$

$g'(x) = 3x^2 \sin^2(4x) + 8x^3 \sin(4x)\cos(4x)$

c. $h(x) = \log_7\left(\dfrac{\sqrt{x^2+3}\ \sin^3 x}{\sqrt[3]{9x}}\right)$

$$= \log_7\left(\sqrt{x^2+3}\right) + \log_7\left(\sin^3 x\right) - \log_7\left(\sqrt[3]{9x}\right)$$

$$= \frac{1}{2}\log_7(x^2+3) + 3\log_7(\sin x) - \frac{1}{3}\log_7(9x)$$

$$h'(x) = \left(\frac{1}{\cancel{2}} \frac{1}{(x^2+3)\ln(7)} \cdot (\cancel{2}x)\right) + \left(\frac{3}{\sin x \cdot \ln 7} \cdot \cos x\right) - \left(\frac{1}{3} \frac{1}{\cancel{9}x \ln 7} \cdot \cancel{9}\right)$$

$$= \frac{x}{(x^2+3)\ln(7)} + \frac{3\cos x}{\sin x \ln 7} - \frac{1}{3x\ln 7}$$

2a. $f(x) = 2(x^2+x^3)^2$

$$f'(x) = 4(x^2+x^3) \cdot (2x+3x^2)$$
$$= 4\left(2x^3 + \underbrace{3x^4 + 2x^4}_{} + 3x^5\right)$$
$$= 4\left(3x^5 + 5x^4 + 2x^3\right) \leftarrow$$
$$= 4x^3\left(3x^2 + 5x + 2\right)$$
$$= 4x^3(3x+2)(x+1)$$

Set $f'(x) = 0$.

$$4x^3(3x+2)(x+1) = 0$$

$x = 0$, $x = -\frac{2}{3}$, $x = -1$

b.

	$x < -1$	$-1 < x < -\frac{2}{3}$	$-\frac{2}{3} < x < 0$	$x > 0$
$4x^3$	$-$	$-$	$-$	$+$
$3x+2$	$-$	$-$	$+$	$+$
$x+1$	$-$	$+$	$+$	$+$
$f'(x)$	$-$	$+$	$-$	$+$

$f(x)$ is increasing on the intervals $(-1, -\frac{2}{3})$ and $(0, \infty)$

$f(x)$ is decreasing on the interval $(-\infty, -1)$ and $(-\frac{2}{3}, 0)$

c. f is decreasing left of $x = -1$ and increasing on the right of $x = -1$. (Concave up) $f(-1) = 0$ is a local minimum.

f is increasing left of $x = -\frac{2}{3}$ and decreasing on the right of $x = -\frac{2}{3}$. (Concave down) $f(-\frac{2}{3})$ is a local maximum.

f is decreasing left of $x = 0$, increasing right of $x = 0$: $f(0) = 0$ is a local minimum.

d. $f''(x) = (15x^4 + 20x^3 + 6x^2)4$

$$= 4x^2(15x^2 + 20x + 6)$$

$f''(-1) = 4 > 0 \Rightarrow$ concave up : local minimum.

$f''(-\frac{2}{3}) = -\frac{32}{27} < 0 \Rightarrow$ concave down : local maximum.

$f''(0) = 0$. Second Derivative Test is inconclusive.

$f''(-0.1) > 0$ cu on left of $x = 0$

$f''(0.1) > 0$ cu on right of zero.

$\Big\}$ local minimum.

3. $f(x) = -3x^3 + 5x + 1$

a. $f'(x) = -9x^2 + 5$ $\qquad f'(0) \neq 0$

$f''(x) = -18x$

Set $f''(x) < 0$

$-18x < 0$

$x > 0$

When $x > 0$ f is concave down

$(0, \infty)$.

Set $f''(x) > 0$

$-18x > 0$

$x < 0$

When $x < 0$ f is concave up.

$(-\infty, 0)$

Set $f''(x) = 0$

$-18x = 0$

$x = 0$

$\uparrow$

Inflection point.

b) $f'(x) = -9x^2 + 5$

Set $f'(x) = 0$

$$-9x^2 + 5 = 0$$
$$x^2 - \frac{5}{9} = 0$$

$x_1 = \frac{\sqrt{5}}{3}$, $x_2 = -\frac{\sqrt{5}}{3}$

$f(-1) = -1$ $\qquad f(1) = 3$

$f(-\frac{\sqrt{5}}{3}) \approx -1.48$

$f(\frac{\sqrt{5}}{3}) \approx 3.48$

Absolute minimum occurs at $(-\frac{\sqrt{5}}{3}, -1.48)$

Absolute maximum occurs at $\cancel{(1, 3)}$. $(\frac{\sqrt{5}}{3}, 3.48)$

4. $(f \circ g)'(1) = ?$

$$(f \circ g)(x) = f(g(x))$$
$$(f \circ g)'(x) = \left(f(g(x))\right)'$$
$$= f'(g(x)) \cdot g'(x)$$

$$(f \circ g)'(1) = f'(g(1)) \cdot g'(1)$$
$$= f'(2) \cdot 6$$
$$= 5 \cdot 6 = 30$$

5. $h(x) = \sqrt{4 + 3f(x)}$, $f(1) = 7$, $f'(1) = 4$, $h'(1) = ?$

$$h(x) = (4 + 3f(x))^{\frac{1}{2}}$$

$$h'(x) = \frac{1}{2}(4 + 3f(x))^{-\frac{1}{2}} \cdot 3 \cdot f'(x)$$

$$= \frac{3f'(x)}{2\sqrt{4 + 3f(x)}}$$

$$h'(1) = \frac{3f'(1)}{2\sqrt{4 + 3f(1)}} = \frac{3 \cdot 4}{2\sqrt{4 + 3(7)}}$$

$$= \frac{12}{10} = \frac{6}{5}$$

6. $f(-3)=0$, $f(0)=3$, $f(2)=1$, $f(3)=0$

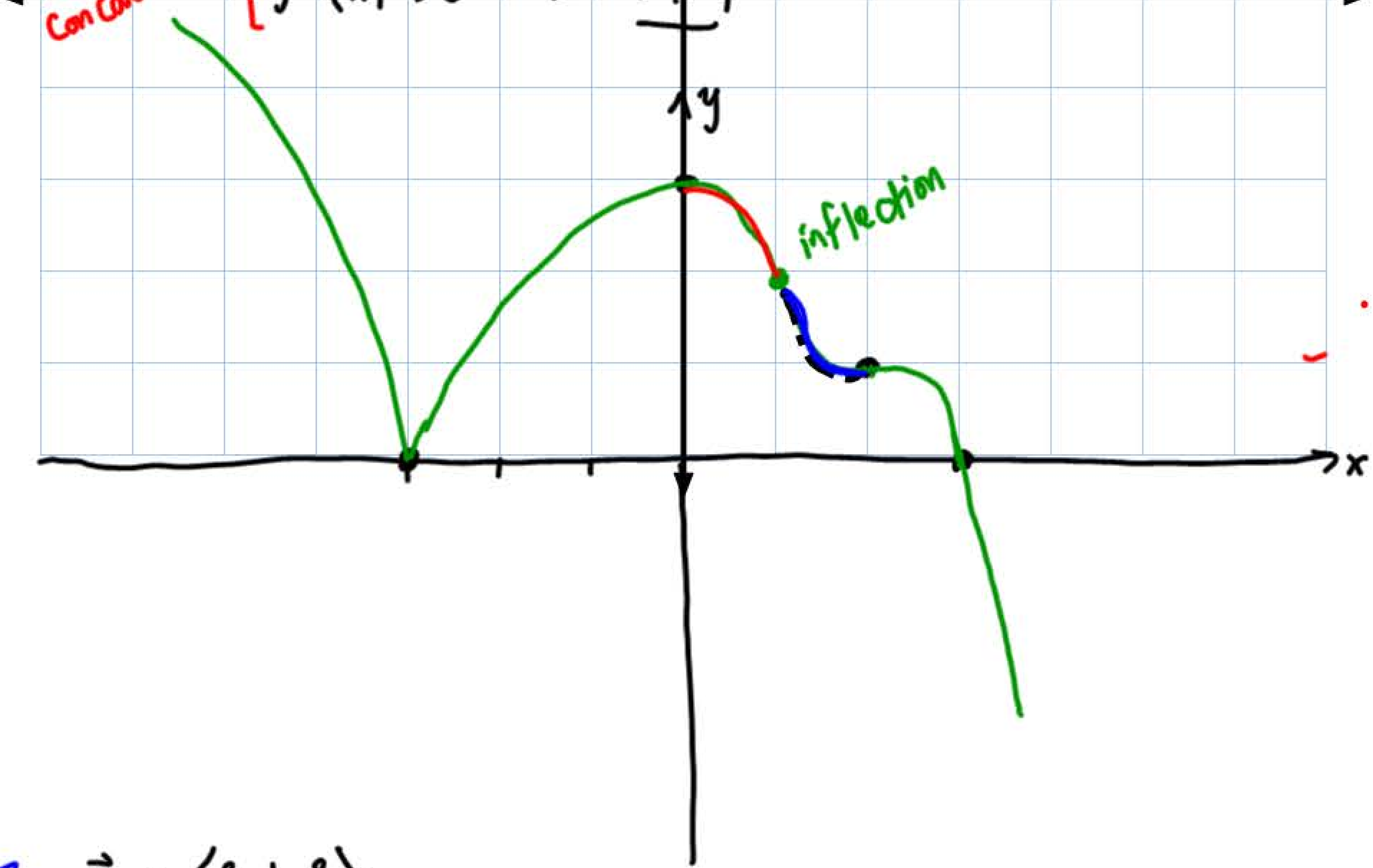
Corner [$f'(-3)$ dne, $f'(0) = f'(2) = 0$

Intervals of decrease [$f'(x) < 0$ on $(-\infty, -3)$, $(0,2)$, $(2,\infty)$

Intervals of increase [$f'(x) > 0$ on $(-3,0)$

Concave down [$f''(x) < 0$ on $(\infty, -3)$, $(-3, 1)$, $(2, \infty)$

Concave up [$f''(x) > 0$ on $(1,2)$



7. $\vec{a} = \langle 3,1,2 \rangle$

$\vec{b} = \langle 7,6,5 \rangle$

$\vec{b} - \vec{a} = \langle 7-3, 6-1, 5-2 \rangle = \langle 4,5,3 \rangle$

$|\vec{b} - \vec{a}| = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{16+25+9} = \sqrt{50} = 5\sqrt{2}$

Since $|\vec{b} - \vec{a}| \neq 1$, $\vec{b} - \vec{a}$ is NOT a unit vector.

Let $\vec{u}$ be the unit vector of $\vec{b} - \vec{a}$ (in the same direction).

$$\vec{u} = \frac{1}{|\vec{b} - \vec{a}|}(\vec{b} - \vec{a}) = \frac{1}{5\sqrt{2}} \langle 4,5,3 \rangle$$

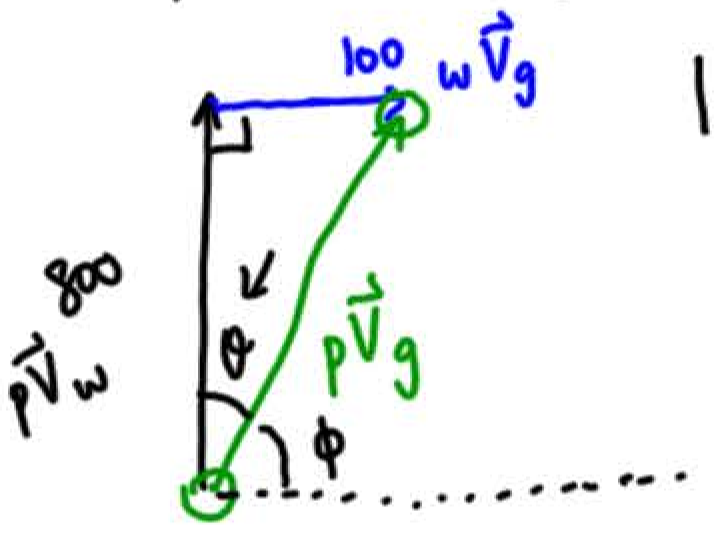
$$= \left\langle \frac{4}{5\sqrt{2}}, \frac{\cancel{5}}{\cancel{5}\sqrt{2}}, \frac{3}{5\sqrt{2}} \right\rangle$$

$$= \left\langle \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{3}{5\sqrt{2}} \right\rangle$$

8. Airplane heading due North 800 km/h.

Wind blowing east at 100 km/h.

$${}_p\vec{V}_g = {}_p\vec{V}_w + {}_w\vec{V}_g$$



$$|{}_p\vec{V}_g| = \sqrt{|{}_p\vec{V}_w|^2 + |{}_w\vec{V}_g|^2}$$

$$= \sqrt{800^2 + 100^2}$$

$$\approx 806.23 \text{ km/hr.}$$

$\theta = \tan^{-1}\left(\frac{800}{100}\right) = \ldots$ relative to positive y-axis (CW)

$\phi = 90 - \theta = \ldots$ relative to positive x-axis. (CCW)

$$f(x) = (\sin x)^{-1}$$

$$g(x) = \sin x$$

$$h(x) = x^{-1}$$

$$f(x) = h(g(x))$$

$$f'(x) = h'(g(x)) \cdot g'(x)$$

$$\frac{df}{dx} = \frac{d((\sin x)^{-1})}{d(\sin x)} \cdot \frac{d(\sin x)}{dx}$$

$$= -(\sin x)^{-2} \cdot (\cos x)$$

$$\frac{dx^2}{dx} = 2x \qquad \frac{dx^{-1}}{dx} = -x^{-2}$$

$$\frac{d((x^2+9)^2)}{d(x^2+9)} = 2(x^2+9)^1$$