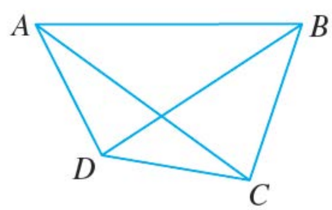
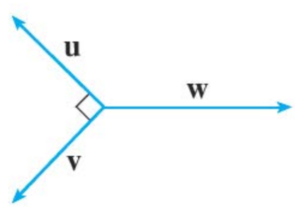
Tutorial 10

Week of November 19, 2018

1. Write the following combinations of vectors as a single vector.
   (a) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$
   (b) $\overrightarrow{CD} + \overrightarrow{DB} = \overrightarrow{CB}$
   (c) $\overrightarrow{DB} - \overrightarrow{AB}$
   $= \overrightarrow{DB} + (-\overrightarrow{AB}) = \overrightarrow{DB} + \overrightarrow{BA} = \overrightarrow{DA}$
   (d) $\overrightarrow{DC} + \overrightarrow{CA} + \overrightarrow{AB}$
   $= \overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DB}$



2. If $|\mathbf{u}| = |\mathbf{v}| = 1$ and $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$, find $|\mathbf{w}|$.

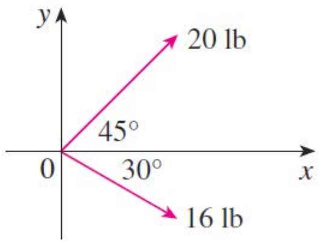


Since $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$, the horizontal components of the vectors should sum to zero and the vertical components of the vectors should sum to zero. The only vectors with a vertical component are $\mathbf{u}$ and $\mathbf{v}$ and it is clear that their vertical components cancel each other out since they run in opposite directions and $|\mathbf{u}| = |\mathbf{v}|$, giving us our zero vertical component sum.

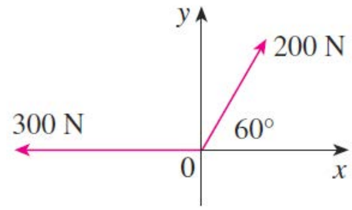
The sum of horizontal components is: $-|\mathbf{u}|\cos(45°) - |\mathbf{v}|\cos(45°) + |\mathbf{w}| = 0$. Rearranging, we obtain:

$$\begin{aligned}
|\mathbf{w}| &= |\mathbf{u}|\cos(45°) + |\mathbf{v}|\cos(45°) \\
&= \cos(45°) + \cos(45°) \\
&= 2\cos(45°) \\
&= 2 \cdot \frac{\sqrt{2}}{2} \\
&= \sqrt{2}
\end{aligned}$$

3. Find the magnitude of the resultant force and the angle it makes with the positive x-axis.



(a)



(b)

(a) Let **a** be the vector of magnitude 20, **b** be the vector of magnitude 16, and **r** be the resultant vector.

$$\begin{aligned} x_{\mathbf{r}} &= x_{\mathbf{a}} + x_{\mathbf{b}} \\ &= 20\cos(45^\circ) + 16\cos(30^\circ) \\ &= 20\frac{\sqrt{2}}{2} + 16\frac{\sqrt{3}}{2} \\ &= 10\sqrt{2} + 8\sqrt{3} \end{aligned}$$

$$\begin{aligned} y_{\mathbf{r}} &= y_{\mathbf{a}} + y_{\mathbf{b}} \\ &= 20\sin(45^\circ) - 16\sin(30^\circ) \\ &= 20\frac{\sqrt{2}}{2} - 16\frac{1}{2} \\ &= 10\sqrt{2} - 8 \end{aligned}$$

$$|\mathbf{r}| = \sqrt{(10\sqrt{2} + 8\sqrt{3})^2 + (10\sqrt{2} - 8)^2} \approx 28.66$$

r is located in the first quadrant since $x_{\mathbf{r}}$ and $y_{\mathbf{r}}$ are positive. Then the angle **r** makes with the positive x-axis is:

$$\theta = \arctan\left(\frac{y_{\mathbf{r}}}{x_{\mathbf{r}}}\right) = \arctan\left(\frac{10\sqrt{2} - 8}{10\sqrt{2} + 8\sqrt{3}}\right) \approx 12.37^\circ$$

Note: arctan is the same thing as $\tan^{-1}$. When putting this into your calculator, make sure your calculator is set to degrees and not radians!

(b) Let **a** be the vector of magnitude 200, **b** be the vector of magnitude 300, and **r** be the resultant vector.

$$\begin{aligned} x_{\mathbf{r}} &= x_{\mathbf{a}} + x_{\mathbf{b}} \\ &= 200\cos(60^\circ) - 300 \\ &= 200\frac{1}{2} - 300 \\ &= -200 \end{aligned}$$

$$\begin{aligned} y_{\mathbf{r}} &= y_{\mathbf{a}} + y_{\mathbf{b}} \\ &= 200\sin(60^\circ) + 0 \\ &= 200\frac{\sqrt{3}}{2} + 0 \\ &= 100\sqrt{3} \end{aligned}$$

$$|\mathbf{r}| = \sqrt{(100\sqrt{3})^2 + (-200)^2} = \sqrt{70000} \approx 264.58$$

$\mathbf{r}$ is located in the second quadrant since $x_{\mathbf{r}}$ is negative and $y_{\mathbf{r}}$ is positive. The angle between $\mathbf{r}$ and the **negative** x-axis is:

$$\theta = \arctan\left(\frac{y_{\mathbf{r}}}{x_{\mathbf{r}}}\right) = \arctan\left(\frac{100\sqrt{3}}{200}\right) \approx 40.89°$$

Then the angle between $\mathbf{r}$ and the **positive** x-axis is $180° - \theta = 180° - 40.89° = 139.11°$.

4. An airplane has an air speed of 300 km/hr and is heading due west. If it encounters a wind blowing south at 50 km/hr, what is the resultant ground velocity of the plane? What is the angle of the resultant ground velocity from the positive x-axis?

   The plane's velocity relative to the ground is the resultant of the sum of the plane speed relative to the wind and the wind speed relative to the ground. In symbols, this is:

   $${}_p\mathbf{v}_g = {}_p\mathbf{v}_w + {}_w\mathbf{v}_g$$

   Since ${}_p\mathbf{v}_w$ is parallel to the x-axis and ${}_w\mathbf{v}_g$ is parallel to the y-axis, the triangle formed is a right triangle in the third quadrant. The resultant vector (${}_p\mathbf{v}_g$) is the hypotenuse of this triangle and we can use Pythagorean Theorem to solve for the magnitude of this vector.

   $$|{}_p\mathbf{v}_g| = \sqrt{(-300)^2 + (-50)^2} \approx 304.138 \text{ km/hr}$$

   The angle it makes with the negative axis is $\theta = \arctan\left(\frac{-50}{-300}\right) \approx 9.46°$ (this is angle below the negative x-axis). The angle from the positive x-axis is $180° + 9.46° = 189.46°$.