Tutorial 11

Week of November 26, 2018

1. Suppose we have two vectors $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ in $\mathbb{R}^3$. Show that for any non-zero scalars $c, d \in \mathbb{R}$:
   (a) $(c\mathbf{a}) \cdot (d\mathbf{b}) = cd\,(\mathbf{a} \cdot \mathbf{b})$
   (b) $(c\mathbf{a}) \times (d\mathbf{b}) = cd\,(\mathbf{a} \times \mathbf{b})$

2. Find $\mathbf{a} \cdot \mathbf{b}$.
   (a) $\mathbf{a} = \langle 5, -2 \rangle, \quad \mathbf{b} = \langle 3, 4 \rangle$
   (b) $\mathbf{a} = \langle 6, -2, 3 \rangle, \quad \mathbf{b} = \langle 2, 5, -1 \rangle$
   (c) $\mathbf{a} = 2\mathbf{i} + \mathbf{j}, \quad \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$
   (d) $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 4\mathbf{i} + 5\mathbf{k}$
   (e) $|\mathbf{a}| = 7, \quad |\mathbf{b}| = 4, \quad \theta = 30°$

3. Find $\mathbf{a} \times \mathbf{b}$. Verify that the cross product is orthogonal to $\mathbf{a}$ and $\mathbf{b}$.
   (a) $\mathbf{a} = \langle 4, 3, -2 \rangle, \quad \mathbf{b} = \langle 2, -1, 1 \rangle$
   (b) $\mathbf{a} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$

4. Find the angles of the triangle given by the points $P(2, 0)$, $Q(0, 3)$, $R(3, 4)$.

5. Using the properties of the cross product, compute the following:
   (a) $\mathbf{k} \times (\mathbf{i} - 2\mathbf{j})$
   (b) $(\mathbf{i} + \mathbf{j}) \times (\mathbf{i} - \mathbf{j})$