Tutorial 4

Week of October 1, 2018

1. Evaluate the following limits, if they exist.

   (a) $\lim\limits_{h \to 0} \dfrac{(-5+h)^2 - 25}{h}$

   (b) $\lim\limits_{u \to 2} \dfrac{\sqrt{4u+1} - 3}{u - 2}$

   (c) $\lim\limits_{t \to 0} \dfrac{\sqrt{1+t} - \sqrt{1-t}}{t}$

   (d) $\lim\limits_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

   (e) $\lim\limits_{x \to -2} \dfrac{2 - |x|}{2 + x}$

2. Explain why $f(x)$ is discontinuous at $x = 0$. Sketch the graph of the function.

$$f(x) = \begin{cases} \cos(x) & x < 0 \\ 0 & x = 0 \\ 1 - x^2 & x > 0 \end{cases}$$

3. Find a and b such that $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & x < 2 \\ ax^2 - bx + 3 & 2 \le x < 3 \\ 2x - a + b & x \ge 3 \end{cases}$$

4. Find the following limit or show it does not exist.

   (a) $\lim\limits_{x \to \infty} \dfrac{3x - 2}{2x + 1}$

   (b) $\lim\limits_{x \to -\infty} \dfrac{x - 2}{x^2 + 1}$

   (c) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{1 + 4x^6}}{2 - x^3}$

   (d) $\lim\limits_{x \to \infty} \left(\ln(1 + x^2) - \ln(1 + x) \right)$