Tutorial 8

Week of November 5, 2018

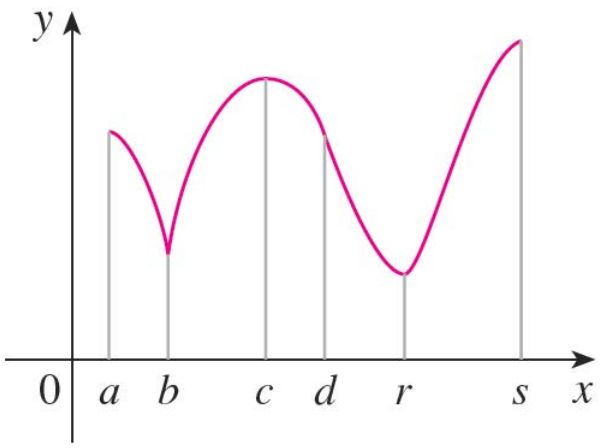
The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geq f(x)$ for all x in some **open** interval around c.
- **local minimum** value of f if $f(c) \leq f(x)$ for all x in some **open** interval around c.

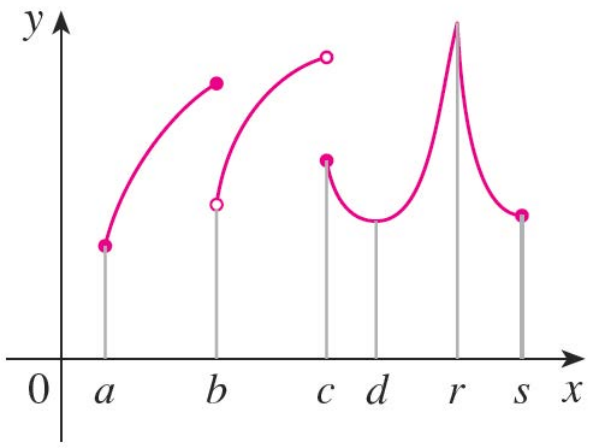
As such, local extrema cannot occur on the endpoints where a function is defined because we cannot find an open interval around that point.

A **critical number** is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist. *Notice that c must be in the domain of f but not f' !!!*

1. For each of the following numbers, a, b, c, d, r, and s, state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum, or neither a maximum nor a minimum.

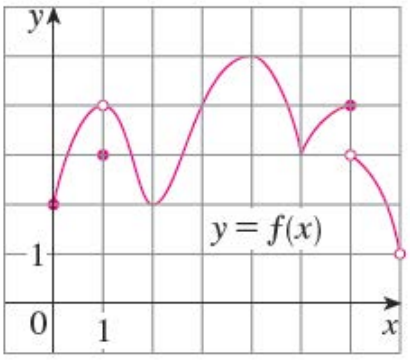


(a)

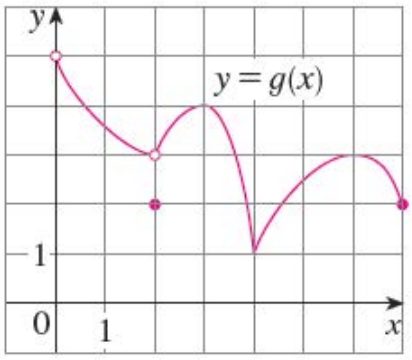


(b)

2. For each of the following graphs, state the points where the local and global extrema occur.



(a)



(b)

3. Find the critical numbers of the given functions.

   (a) $f(x) = x^3 + 6x^2 - 15x$

   (b) $f(x) = 2x^3 - 3x^2 - 36x$

   (c) $g(t) = |3t - 4|$

   (d) $h(p) = \dfrac{p-1}{p^2+4}$

   (e) $g(x) = \sqrt[3]{4 - x^2}$

4. Find the absolute maximum and minimum of the following functions on the given interval.

   (a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 1, \quad [-2, 3]$

   (b) $f(t) = (t^2 - 4)^3, \quad [-2, 3]$

   (c) $f(x) = \dfrac{x}{x^2 - x + 1}, \quad [0, 3]$

5. For each of the following functions:

   (i) Find the intervals of increase and decrease

   (ii) Find the values of the local maximum and minimum

   (iii) Find the intervals of concavity and inflection points

   (a) $f(x) = x^3 - 3x^2 - 9x + 4$

   (b) $f(x) = 2x^3 - 9x^2 + 12x - 3$

   (c) $f(x) = x^2 \ln x$

6. For the following function, find:

   (a) The vertical and horizontal asymptotes

   (b) The intervals of increase and decrease

   (c) The values of the local maximum and minimum

   (d) The intervals of concavity and inflection points

$$f(x) = e^{-x^2}$$