Tutorial 2

Question 1

(9.78) Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a power family distribution with parameters α and $\theta = 3$. Then for $\alpha > 0$,

$$f(y \mid \alpha) = \begin{cases} \dfrac{\alpha y^{\alpha-1}}{3^\alpha}, & 0 \le y \le 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that $\mathbf{E}(Y_1) = 3\alpha/(\alpha+1)$.

(b) Derive the method of moments estimator for α.

Question 2

(9.84) A certain type of electronic component has a lifetime Y (in hours) with probability density function given by:

$$f(y \mid \theta) = \begin{cases} \dfrac{1}{\theta^2} y e^{-y/\theta}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

That is, Y has a gamma distribution with parameters $\alpha = 2$ and θ. Let $\hat{\theta}$ denote the MLE of θ. Suppose that three such components, tested independently, had lifetimes (in hours):

$$120 \quad 130 \quad 128.$$

(a) Find the MLE of θ and provide an estimate using the given data.

(b) Find $\mathbf{E}(\hat{\theta})$ and $\mathbf{Var}(\hat{\theta})$. Is $\hat{\theta}$ an unbiased estimator of θ?

(c) What is the MLE for the variance of Y?

Question 3

Generate an observation from the binomial distribution with $p = 0.4$ and $n = 40$.

(a) Find a 90% confidence interval for p, assuming that you did not know the true value of p.

(b) Consider the interval from (a). Construct 200 such intervals based on a random sample of size $m = 200$ from the Binomial($n = 40$, $p = 0.4$) distribution.

(c) How many of your intervals contained the true value of p? Was this expected or unexpected?