# Tutorial 3

## Question 1

(10.2) An experimenter has prepared a drug dosage level that she claims will induce sleep for 80% of people suffering from insomnia. After examining the dosage, we feel that her claims regarding the effectiveness of the dosage are inflated. In an attempt to disprove her claim, we administer her prescribed dosage to 20 insomniacs and we observe $Y$, the number for whom the drug dose induces sleep. We wish to test the hypotheses:

$$H_0 : p = 0.8 \quad \text{vs} \quad H_1 : p < 0.8.$$

Assume that the rejection region $\{y \leq 12\}$ is used.

(a) In the context of this problem, what is a type I error?

(b) Find $\alpha$, the probability of committing a type I error.

(c) In the context of this problem, what is a type II error?

(d) Find $\beta$, the probability of committing a type II error, when $p = 0.6$.

(e) Find $\beta$, the probability of committing a type II error, when $p = 0.4$.

## Question 2

(10.94) Suppose that $Y_1, Y_2, \ldots, Y_n$ constitute a random sample from a normal distribution with known mean $\mu$ and unknown variance $\sigma^2$. Find the most powerful $\alpha$-level test of

$$H_0 : \sigma^2 = \sigma_0^2 \quad \text{vs} \quad H_1 : \sigma^2 = \sigma_1^2,$$

where $\sigma_1^2 > \sigma_0^2$. Show that this test is equivalent to a $\chi^2$ test.

## Question 3

Generate $n = 30$ observations, $X_1, X_2, \ldots, X_n$, from a Bernoulli distribution with parameter $p = 0.4$. Consider the problem of testing

$$H_0 : p = 0.3 \quad \text{vs.} \quad H_1 : p > 0.3,$$

by means of a test that rejects $H_0$ for *large* values of the test statistic

$$T(X_1, X_2, \ldots, X_n) = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{\hat{p} - 0.3}{\sqrt{0.3(1 - 0.3)/n}},$$

where $\hat{p} = \frac{1}{n} \sum_{i=1}^{n} X_i$ is the sample proportion.

(a) Do you reject $H_0$ at level $\alpha = 0.05$?

(b) Find the (approximate) $p$-value. Hint: use the fact that, in view of the CLT, when $p = 0.3$,

$$T(X_1, X_2, \dots, X_n) \overset{\text{approx}}{\sim} N(0,1),$$

so that the $p$-value is given by

$$\mathbf{P}_{p=0.3}\left(T(X_1, X_2, \dots, X_n) \geq T(x_1, x_2, \dots, x_n)\right) \approx \mathbf{P}\left(N(0,1) \geq T(x_1, x_2, \dots, x_n)\right),$$

where $T(x_1, x_2, \dots, x_n)$ is the observed value of the test statistic.