Tutorial 8

Question 1

Specimens of milk from a number of dairies in three different districts were analysed, and the concentration of the radioactive isotope strontium-90 was measured in each specimen. Suppose that specimens were obtained from 4 dairies in the first district, 6 dairies in the second district, and 3 dairies in the third district, measured in picocuries per litre.

District (i)	n_i	Concentration
1	4	6.4
		5.8
		6.5
		7.7
2	6	7.1
		9.9
		11.2
		10.5
		6.5
		8.8
3	3	9.5
		9.0
		12.1

(a) Assuming that the variance of the concentration of strontium-90 is the same for the dairies in all three districts, determine the maximum likelihood estimate of the mean concentration in each of the districts and the maximum likelihood estimate of the common variance.

(b) Test the hypothesis that the three districts have identical concentrations of strontium-90.

Question 2

Show that in a one-way layout, the following statistic is an unbiased estimator of σ^2:

$$\frac{1}{n-k}\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij}-\overline{Y}_{i\bullet})^2,$$

where

$$\overline{Y}_{i\bullet} = \frac{1}{n_i}\sum_{j=1}^{n_i}Y_{ij}.$$

Question 3

(13.8) In a study of starting salaries for assistant professors, five male assistant professors at each of three types of doctoral-granting institutions were randomly polled and their starting salaries were recorded under the condition of anonymity. The results of the survey (measured in $1000) are given in the following table.

Public Universities	Private-Independent	Church-Affiliated
49.3	81.8	66.9
49.9	71.2	57.3
48.5	62.9	57.7
68.5	69.0	46.2
54.0	69.0	52.2

(a) ℝ Is there sufficient evidence to indicate a difference in the average starting salaries of assistant professors at the three types of doctoral-granting institutions? Use the rejection region method.

(b) ℝ Repeat the above using the p-value method.

Question 4

(13.10) An incredibly long story that I don't want to re-write.

Method A	Method B	Method C
73	54	79
83	74	95
76	71	87
68		
80		

(a) ℝ Do the data provide sufficient evidence to indicate that at least one of the methods of treatment produces a mean student response different from the other methods? Use the rejection region method at the 5% level of significance.

(b) ℝ Repeat the above using the p-value method.