Stat 2605 Tutorial 1

September 20, 2022

1. Toss a die twice and record the sum. Find the probability that the sum is 10.

   The results of the die tosses can be represented as ordered pairs of the form (i, j), with i being the outcome of the first toss, and j being the outcome of the second toss, and $1 \leq i, j \leq 6$.

   The ordered pairs that will result in a sum of 10 are $(4, 6)$, $(5, 5)$, and $(6, 4)$.

   There are $6 * 6 = 36$ possible ordered pairs.

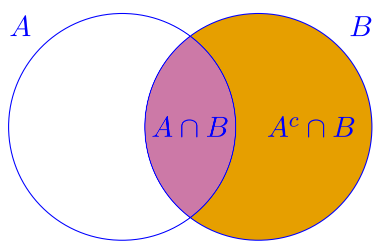
   The probability of obtaining a sum of 10 is 3/36, which can be reduced to 1/12.

2. Suppose $\mathbf{P}(A) = 0.4$, $\mathbf{P}(B) = 0.8$, $\mathbf{P}(A \cap B) = 0.3$. Find $\mathbf{P}(A^c \cup B)$.

   By the inclusion-exclusion principle,

$$\mathbf{P}(A^c \cup B) = \mathbf{P}(A^c) + \mathbf{P}(B) - \mathbf{P}(A^c \cap B). \qquad (\star)$$

   But how can we find $\mathbf{P}(A^c \cap B)$?



   Through the use of Venn diagrams (above), we can identify the region that is $A^c \cap B$. Furthermore, we can see that the set B can be expressed as the disjoint union of the sets $A \cap B$ and $A^c \cap B$, i.e.

$$B = (A \cap B) \cup (A^c \cap B).$$

   Recall that one of the axioms of probability stated that the probability of a union of disjoint sets was equal to the sum of the probabilities of the disjoint sets, i.e.

$$\mathbf{P}(B) = \mathbf{P}((A \cap B) \cup (A^c \cap B)) = \mathbf{P}(A \cap B) + \mathbf{P}(A^c \cap B).$$

   Rearranging, we obtain

$$\mathbf{P}(A^c \cap B) = \mathbf{P}(B) - \mathbf{P}(A \cap B),$$

which can we can substitute back into ($\star$).

$$
\begin{aligned}
\mathbf{P}(A^c \cup B) &= \mathbf{P}(A^c) + \mathbf{P}(B) - \mathbf{P}(A^c \cap B) \\
&= \mathbf{P}(A^c) + \mathbf{P}(B) - (\mathbf{P}(B) - \mathbf{P}(A \cap B)) \\
&= \mathbf{P}(A^c) + \mathbf{P}(A \cap B) \\
&= 1 - \mathbf{P}(A) + \mathbf{P}(A \cap B) \\
&= 1 - 0.4 + 0.3 \\
&= 0.9
\end{aligned}
$$

3. A class consists of seven boys and eight girls. Four students are selected at random to volunteer on campus.

   (a) How many ways we can select four students such that there is *at least* one girl?

   Direct method: Consider the cases where there is one girl, two girls, three girls, and four girls.

   One girl: $\binom{7}{3} * \binom{8}{1} = 280$

   Two girls: $\binom{7}{2} * \binom{8}{2} = 588$

   Three girls: $\binom{7}{1} * \binom{8}{3} = 392$

   Four girls: $\binom{7}{0} * \binom{8}{4} = 70$

   Therefore, the number of ways that we can select four students such that there is at least one girl is

   $$280 + 588 + 392 + 70 = 1330.$$

   (b) What is the probability of getting such a team?

   The total number of possible teams of four students is computed as $\binom{15}{4} = 1365$.

   The probability of getting a team of four students where at least one of the students is a girl is 1330/1365, which can be reduced to 266/273.

   Exercise: Solve (a) using the indirect method.