Stat 2605 Tutorial 6

November 15, 2022

1. Suppose (X, Y) has joint pmf given by:

f(x,y)			\mathbf{x}	
		1	2	3
	0	0.2	0.1	0.1
\mathbf{y}	1	0.1	0.3	0
	2	0	0.1	0.1

and zero otherwise. Find $\mathbf{E} (2X^2Y + 1)$.

2. Suppose (X, Y) has joint pmf given by:

f(x,y)			\mathbf{x}	
		2	3	4
	-1	0.1	0.2	0.05
\mathbf{y}	0	0.27	0.1	0.15
	1	0	0.03	0.1

and zero otherwise.

- (a) Are X and Y independent?
- (b) Show that $\mathbf{E}(XY) \neq \mathbf{E}(X)\mathbf{E}(Y)$.
- 3. Suppose (X, Y) has joint pdf given by:

$$f(x,y) = \begin{cases} \frac{x+y}{3} & 0 < x < 2, \quad 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the marginal pdfs $f_X(x)$ and $f_Y(y)$.
- (b) Are X and Y independent?
- (c) Find the conditional mean $\mathbf{E}\left(Y\,|\,X=1\right)$.
- 4. Suppose that the distribution of the lifetime of a light bulb has an exponential distribution with a mean of 900 hours. Find the probability that the total lifetime of 20 bulbs exceeds 22000 hours.

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5. Suppose the random variable, X, has a Geometric (p) distribution, with pmf given by:

$$f(x) = \begin{cases} (1-p)^{x-1}p & x = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

Find the mgf, $M_X(t)$, and use it to find $\mathbf{E}\left(X\right)$.