Stat 2605 Tutorial 7

November 22, 2022

1. Suppose X and Y are independent with:

$$\mathbf{E}\left(X\right) = 2, \quad \mathbf{E}\left(Y\right) = 3, \quad \mathbf{Var}\left(X\right) = 1, \quad \mathbf{Var}\left(Y\right) = 2$$

   Let $Z = 2X - Y + 1$. Use Chebyshev's inequality to find a lower bound for $\mathbf{P}\left(|Z - 2| < 5\right)$.

2. Suppose check-ins at a small hotel occur satisfying the three conditions of a Poisson process, $N(t)$. The rate is $\lambda = 6$ and the unit of time is an hour. Find the probability that there are more than 2 check-ins within 15 minutes.

3. Continuing from the setting of the previous problem, let T_2 and T_3 be inter-arrival times, where T_2 is the time between first and second arrivals, and T_3 is the time between second and third arrivals. Determine the joint pdf of T_2 and T_3. Find $\mathbf{Var}\left(T_2 + T_3\right)$.

4. Suppose X and Y are independent exponential random variables with:

$$\mathbf{E}\left(X\right) = 2, \quad \mathbf{E}\left(Y\right) = 3$$

   (a) Compute $\mathbf{E}\left(XY\right)$.

   (b) Compute $\mathbf{E}\left(X^2Y^2\right)$.

   (c) Compute $\mathbf{E}\left(X^2Y\right)$.