Tutorial 3: Questions

January 31, 2018

Review

Let X_1, X_2, …, X_n be an iid sample from a distribution with mean μ and variance σ^2.

- In general, $\mathbf{E}(\overline{X}) = \mu$, and $\mathbf{Var}(\overline{X}) = \sigma^2/n$. Similarly, $\mathbf{E}(T) = n\mu$, and $\mathbf{Var}(T) = n\sigma^2$.
- In the special case that these X_is come from a normal distribution, then:
  - $\overline{X} \sim N(\mu, \sigma^2/n)$ and
  - $T \sim N(n\mu, n\sigma^2)$.

Question 5.4.54, Page 237

Suppose the sediment density (g/cm) of a randomly selected specimen from a certain region is normally distributed with mean 2.65 and standard deviation 0.85.

(a) If a random sample of 25 specimens is selected, what is the probability that the sample average sediment density is at most 3.00? Between 2.65 and 3.00?

(b) How large a sample size would be required to ensure that the first probability in part (a) is at least .99?

Question 5.5.72, Page 243

I have three errands to take care of in the Administration Building. Let X_i be the time that it takes for the i^{th} errand $(i = 1, 2, 3)$, and let X_4 be the total time in minutes that I spend walking to and from the building and between each errand. Suppose the X_is are independent and normally distributed, with the following means and standard deviations:

$$\mu_1 = 15,\ \sigma_1 = 4; \quad \mu_2 = 5,\ \sigma_2 = 1; \quad \mu_3 = 8,\ \sigma_3 = 2; \quad \mu_4 = 12,\ \sigma_4 = 3.$$

I plan to leave my office at precisely 10:00 a.m. and wish to post a note on my door that reads, "I will return by t a.m." What time t should I write down if I want the probability of my arriving after t to be 0.01?

Question 6.1.10, Page 263

Using a long rod that has length μ, you are going to lay out a square plot in which the length of each side is μ. Thus the area of the plot will be μ^2. However, you do not know the value of μ, so you decide to make n independent measurements $X_1, X_2, \ldots, X_n$ of the length. Assume that each X_i has mean μ (unbiased measurements) and variance σ^2.

(a) Show that $\overline{X}^2$ is not an unbiased estimator for μ^2. [Hint: For any rv Y, $\mathbf{E}(Y^2) = \mathbf{Var}(Y) + \mathbf{E}(Y)^2$. Apply this with $Y = \overline{X}$.

(b) For what value of k is the estimator $\overline{X}^2 - kS^2$ unbiased for μ^2? [Hint: Compute $\mathbf{E}(\overline{X}^2 - kS^2)$.]

Finding a MLE: Exponential

Suppose we are given a sample $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \text{Exp}(\theta)$, where θ is an unknown parameter ($\theta > 0$). Find a Maximum Likelihood Estimate for θ. Is the maximum likelihood estimator that you obtain also an unbiased estimator?