

Tutorial 6

February 28, 2018

- Today's tutorial is mostly going to be a Q & A session with only a few review problems that I would like to go over

Question 1

Suppose we have some random sample X_1, X_2, \dots, X_n from some unknown distribution with mean μ and variance σ^2 .

- (a) Find the mean and variance of the random variable \bar{X} .
- (b) Show that the sample variance,

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

can also be written as

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n X_i^2 - n\bar{X}^2 \right)$$

- (c) Is the sample variance, S^2 , an unbiased estimator of the population variance σ^2 ?
- (d) Go over what we mean when we say unbiased estimator. When we have multiple unbiased estimators we prefer ones with minimum variance. Example of a biased estimator.

Question 2

- (a) Properties of Expected Value and Variance. Pitfalls of general formula for expected value and variance of \bar{X} . Go over other misconceptions.
- (b) Suppose we have a random sample of luggage weights of economy class passengers and first class passengers. The mean and standard deviation of economy class luggage is 40 lbs and 10 lbs, respectively, and the mean and standard deviation of first class passengers is 30 lbs and 6 lbs, respectively.
 - (i) If we have 12 first class passengers and 50 economy class passengers on a single flight, what is the mean and variance of the total luggage weight?
 - (ii) What is the mean and variance of the sample mean weight?
 - (iii) Suppose we define a new random variable that involves adding 4 times an economy luggage weight and subtracting 3 times a first class luggage weight. What is the mean and variance of this new variable if we still have 12 first class and 50 economy passengers?