Tutorial 8: Solutions

March 14, 2018

Question 8.1.14, Page 326

A new design for the braking system on a certain type of car has been proposed. For the current system, the true average braking distance at 40 mph under specified conditions is known to be 120 ft. It is proposed that the new design be implemented only if sample data strongly indicates a reduction in true average braking distance for the new design.

(a) Define the parameter of interest and state the relevant hypotheses.

The parameter of interest is μ, the true average braking distance at 40 mph. Our hypotheses are $H_0 : \mu \geq 120$, $H_A : \mu < 120$.

(b) Suppose braking distance for the new system is normally distributed with $\sigma = 10$. Let $\overline{X}$ denote the sample average braking distance for a random sample of 36 observations. Which values of $\overline{x}$ are more contradictory to H_0 than 117.2, what is the p-value in this case, and what conclusion is appropriate if $\alpha = 0.10$?

Let X_i be the braking distance for the new system. Then $X_i \sim N(\mu = 120, \sigma^2 = 10^2)$. It follows that $\overline{X} \sim N(\mu_{\overline{X}} = 120, \sigma_{\overline{X}}{}^2 = 10^2/36)$. Since we are performing a lower-tailed test, values of $\overline{x}$ that are more contradictory than 117.2 (under H_0) would be values that are less than 117.2.

Transforming this value to standard normal:

$$z = \frac{117.2 - 120}{10/6} = -1.68$$

Since this is a lower-tailed test, the p-value is the area to the left of z. $\Phi(-1.68) = 0.04648$.

It is given that $\alpha = 0.10$. We reject H_0 if $p < \alpha$. Since $0.04648 < 0.10$, we can reject H_0 at the 10% significance level and conclude that the data suggests a reduction in braking distance with the new system.

(c) What is the probability that the new design is not implemented when its true average braking distance is actually 115 ft and the test from part (b) is used?

Let the CRB (Critical Region Bound) be 117.2.

$$\begin{aligned}
\beta(115) &= \mathbf{P}\left(\text{Type II Error} \mid \mu' = 115\right) \\
&= \mathbf{P}\left(\text{Fail to reject } H_0 \text{ when } H_0 \text{ untrue}\right) \\
&= \mathbf{P}\left(\overline{X} > \text{CRB} \mid \mu' = 115\right) \\
&= \mathbf{P}\left(Z > \frac{\text{CRB} - \mu'}{\sigma/\sqrt{n}}\right) \\
&= 1 - \mathbf{P}\left(Z < \frac{\text{CRB} - \mu'}{\sigma/\sqrt{n}}\right)
\end{aligned}$$

$$= 1 - \mathbf{P}\left(Z < \frac{117.2 - 115}{10/6}\right)$$

$$= 0.09342$$

Notice that this formula is somewhat different compared to last week's formula (8.1.12c) and the one in the textbook on page 331. This is because last time we had to solve for our critical region. This time it was given. We recall from last time that when $H_A : \mu > \mu_0$, we had

$$\begin{aligned} & \mathbf{P}\left(Z < z_{1-\alpha} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) \\ = & \mathbf{P}\left(Z < \frac{\left(z_{1-\alpha}\frac{\sigma}{\sqrt{n}} + \mu_0\right) - \mu'}{\sigma/\sqrt{n}}\right) \\ = & \mathbf{P}\left(Z < \frac{\text{CRB} - \mu'}{\sigma/\sqrt{n}}\right) \end{aligned}$$

It follows that using the CRB where appropriate will give an equivalent formulation for finding Type II Error.

Question 8.4.52, Page 352

In a sample of 171 students at an Australian university that introduced the use of plagiarism-detection software in a number of courses, 58 students indicated a belief that such software unfairly targets students. Does this suggest that a majority of students at the university do not share this belief? Test appropriate hypotheses.

We are given that $x = 58$, $n = 171$. Since we want to test whether the majority of students **DO NOT** share this belief, we obtain the hypotheses: $H_0 : p \geq 0.50$, $H_A : p < 0.5$.

$np_0 = (171)(0.50) = 85.5 \geq 10$
$n(1 - p_0) = (171)(0.50) = 85.5 \geq 10$
We can proceed with a large sample hypothesis test for proportions.

$$z = \frac{\frac{58}{171} - 0.50}{\sqrt{0.5(1 - 0.5)/171}} = -4.205$$

Taking $\alpha = 0.05$, since this is a lower-tailed test, we obtain $-z_{1-\alpha} = -z_{0.95} = -1.6449$. We reject H_0 if $z < -z_{1-\alpha}$. Since $-4.206 < -1.6449$, we reject H_0 . We conclude at the 5% level of significance that this belief is not shared by the majority of students.

Question 8.S.66, Page 358

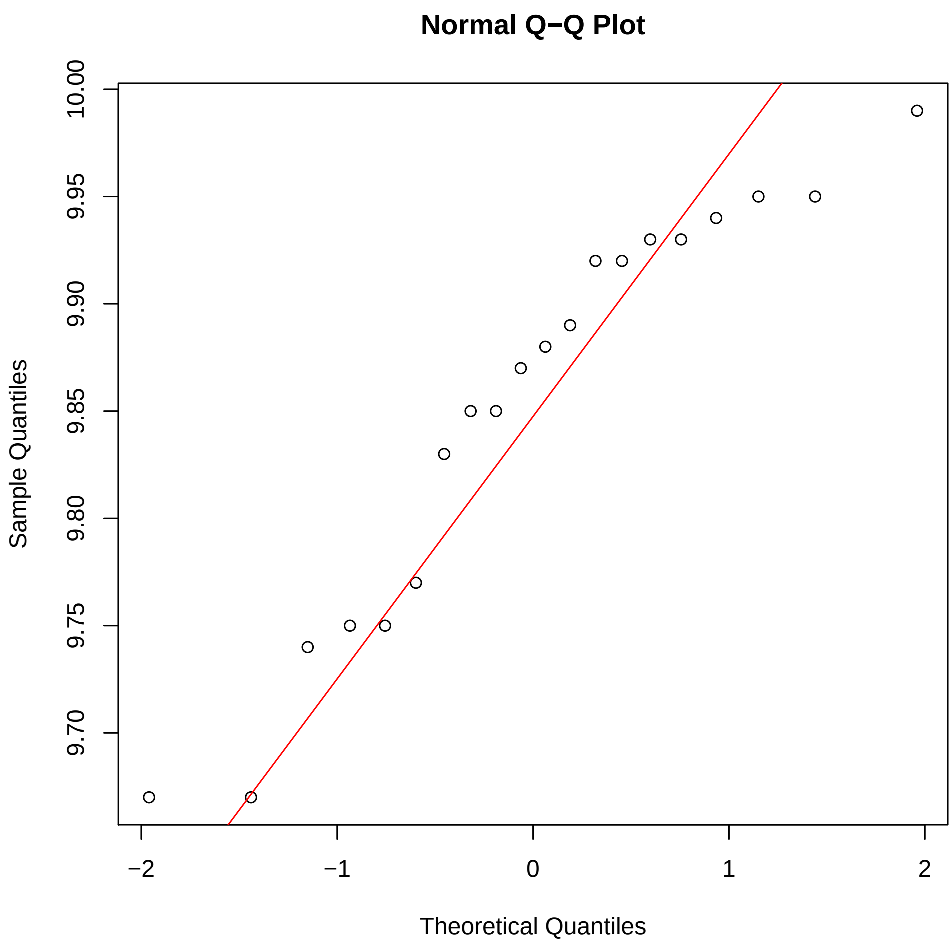
The accompanying observations on residual flame time (seconds) for strips of treated children's nightwear were given in an article. Suppose a true average flame time of at most 9.75 had been mandated. Does the data suggest that this condition has not been met? Carry out an appropriate test after first investigating the plausibility of assumptions that underlie your method of inference.

9.85	9.93	9.75	9.77	9.67	9.87	9.67
9.94	9.85	9.75	9.83	9.92	9.74	9.99
9.88	9.95	9.95	9.93	9.92	9.89	

In order to formulate our hypotheses, the keywords to observe here are "**at most 9.75**" and "does the data suggest condition has **not** been met". Our hypotheses are $H_0 : \mu \leq 9.75$, $H_A : \mu > 9.75$.

- $n = 20$, σ unknown, so we must approximate σ with s
- Due to all this extra variability, we must use Student's t-distribution
- What are the conditions to use Student's t-distribution? The underlying distribution must be normal.
- We verify this in `R` by first importing our data into a variable called `flame`. Then:

```
qqnorm(flame)
qqline(flame, col = "red")
```



Normality seems like a reasonable assumption so we proceed with calculating the test statistic.

$\overline{x} = 9.8525$, $s = 0.096457...$ (use the exact value in your calculations).

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{9.8525 - 9.75}{0.096457.../\sqrt{20}} = 4.7523$$

This is an upper-tailed test so we require $t_{n-1,\alpha} = t_{19,0.05} = 1.729$. We reject H_0 if $t > t_{19,0.05}$. Since $4.7523 > 1.729$, we can reject H_0 . We conclude at the 5% level of significance that the data suggests the required condition has not been met.

Mass of Pizza Pockets

Suppose it is known that pizza pockets across brands are normally distributed with population standard deviation 20. In addition, it is known that brand-name pizza pockets have a mean of 100 grams. A researcher is interested in seeing if store-brand pizza pockets have less mass than brand-name pockets. The researcher obtains a sample of 20 store-brand pizza pockets and records that the average mass of the pizza pockets is 91.65.

(a) Test the appropriate hypotheses using the p-value method (assume the level of significance is 5%).

From the question we are given $\mu = 100$, $\sigma = 20$, $n = 20$. Since we are testing whether store-brand pizza pockets have less than name brands, we obtain the hypotheses:

$H_0 : \mu \geq 100$, $H_A : \mu < 100$.

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{91.65 - 100}{20/\sqrt{20}} = -1.867$$

Since this is a lower-tailed test, the p-value of our test statistic is the area to the left. As such we can plug this directly into the Phi function.

$$p = \Phi(-1.867) = 0.03095$$

We reject H_0 if $p < \alpha$. Since $0.03095 < 0.05$ we reject H_0 and conclude that store-brand pizza pockets have less mass compared to brand-name pizza pockets.

(b) What is the power of a test at the 5% level of significance if the true mean mass of store-brand pizza pockets is 95?

We start by find the probability of committing a Type II Error.

$$\begin{aligned}
\beta(95) &= \mathbf{P}\left(\text{Type II Error} \mid \mu' = 95\right) \\
&= \mathbf{P}\left(\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > -z_{1-\alpha} \middle| \mu' = 95\right) \\
&= \mathbf{P}\left(\overline{X} > -z_{1-\alpha}\frac{\sigma}{\sqrt{n}} + \mu_0 \middle| \mu' = 95\right) \\
&= \mathbf{P}\left(\frac{\overline{X} - \mu'}{\sigma/\sqrt{n}} > -z_{1-\alpha} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) \\
&= \mathbf{P}\left(Z > -z_{1-\alpha} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) \\
&= 1 - \mathbf{P}\left(Z < -1.6449 + \frac{100 - 95}{20/\sqrt{20}}\right) \\
&= 1 - \mathbf{P}\left(Z < -0.5269\right)
\end{aligned}$$

$$
\begin{aligned}
&= 1 - 0.29914 \\
&= 0.70086
\end{aligned}
$$

The power of a hypothesis test is the probabiliy of correctly rejecting the null hypothesis in favour of the alternative hypothesis when the alternative hypothesis is actually true.

$$
\begin{aligned}
\text{Power} &= 1 - \beta(95) \\
&= 1 - 0.70086 \\
&= 0.29914
\end{aligned}
$$

Remember that a higher power is better. Unfortunately, we have a very low power here.