Tutorial 3

February 6, 2020

Question 1

Compute the following probabilities.

(a) $b(3;\ 8,\ 0.35)$

This tells us that $X \sim \text{Binom}(n = 8,\ p = 0.35)$ and we are looking for $\mathbf{P}\left(X = 3\right)$.

$$\mathbf{P}\left(X = 3\right) = \binom{8}{3}(0.35)^3(0.65)^5$$

$$= 0.2786$$

(b) $b(5;\ 8,\ 0.6)$

This tells us that $X \sim \text{Binom}(n = 8,\ p = 0.6)$ and we are looking for $\mathbf{P}\left(X = 5\right)$.

$$\mathbf{P}\left(X = 5\right) = \binom{8}{5}(0.6)^5(0.4)^3$$

$$= 0.2787$$

(c) $\mathbf{P}\left(3 \leq X \leq 5\right)$, where $X \sim \text{Binom}(n = 7,\ p = 0.6)$

$$\begin{aligned}\mathbf{P}\left(3 \leq X \leq 5\right) &= \mathbf{P}\left(X = 3\right) + \mathbf{P}\left(X = 4\right) + \mathbf{P}\left(X = 5\right) \\ &= \binom{7}{3}(0.6)^3(0.4)^4 + \binom{7}{4}(0.6)^4(0.4)^3 + \binom{7}{5}(0.6)^5(0.4)^2 \\ &= 0.7451\end{aligned}$$

Question 2

A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified "seconds". Suppose that six goblets are randomly selected.

Let X represent the number of flawed goblets in a random sample of six goblets. Then:

$$X \sim \text{Binom}(n = 6,\, p = 0.10)$$

(a) How likely is it that only one is a second?

$$\begin{aligned}\mathbf{P}(X = 1) &= \binom{6}{1}(0.1)^1(0.9)^5 \\ &= 0.3543\end{aligned}$$

(b) What is the probability that at least two are seconds?

$$\begin{aligned}\mathbf{P}(X \geq 2) &= \mathbf{P}(X = 2) + \mathbf{P}(X = 3) + \ldots + \mathbf{P}(X = 6) \\ &= 1 - \mathbf{P}(X < 2) \\ &= 1 - \mathbf{P}(X \leq 1) \\ &= 1 - \big(\mathbf{P}(X = 1) + \mathbf{P}(X = 0)\big) \\ &= 1 - 0.3543 - \binom{6}{0}(0.1)^0(0.9)^6 \\ &= 0.1143\end{aligned}$$

(c) What is the average number of the goblets that are second?

If X has a $\text{Binom}(n, p)$ distribution, then $\mathbf{E}(X) = np$.

$$\mathbf{E}(X) = np = (6)(0.1) = 0.6$$

Question 3

An electronics store has received a shipment of 20 table radios that have connections for iPod or iPhone. 12 of these have two slots (so they can accommodate both devices), and the other 8 have a single slot. Suppose that 6 of the 20 radios are randomly selected to be stored under a shelf where the radios are displayed, and the remaining ones are placed in a storeroom. Let X be the number, among the radios stored under display shelf, that have two slots.

(a) What kind of distribution does X have?

We have a finite population ($N = 20$) from which we are drawing a sample ($n = 6$). We are interested in observing the number of two slot radios selected in this sample. There are 12 two slot radios ($M = 12$) within this population. Let X represent the number of two slot radios selected from a sample of 6. Then:

$$X \sim \text{Hypergeometric}(N = 20,\, M = 12,\, n = 6)$$

(b) Compute:

i. $\mathbf{P}(X = 2)$

$$\mathbf{P}(X = 2) = \frac{\overbrace{\binom{12}{2}}^{\text{successes}} \overbrace{\binom{8}{4}}^{\text{failures}}}{\underbrace{\binom{20}{6}}_{\text{samples of size 6}}}$$

$$= 0.1192$$

ii. $\mathbf{P}(X \leq 2)$

$$\mathbf{P}(X \leq 2) = \mathbf{P}(X = 0) + \mathbf{P}(X = 1) + \mathbf{P}(X = 2)$$

$$= \frac{\binom{12}{0}\binom{8}{6} + \binom{12}{1}\binom{8}{5} + \binom{12}{2}\binom{8}{4}}{\binom{20}{6}}$$

$$= 0.1373$$

iii. $\mathbf{P}(X \geq 2)$

$$\begin{aligned}
\mathbf{P}(X \geq 2) &= 1 - \mathbf{P}(X < 2) \\
&= 1 - \mathbf{P}(X \leq 1) \\
&= 1 - \big(\mathbf{P}(X = 0) + \mathbf{P}(X = 1)\big) \\
&= \frac{\binom{20}{6} - \binom{12}{0}\binom{8}{6} - \binom{12}{1}\binom{8}{5}}{\binom{20}{6}} \\
&= 0.9819
\end{aligned}$$

Question 4

An interviewer is conducting a phone survey. Each call has probability 0.4 to be answered. What is the probability that the interviewer has to try:

(a) Three calls before getting the first call answered?

Note that we deviate slightly from the textbook in how we define the geometric distribution. (Compare example 3.12 on page 103 to your class notes)

Let X represent the number of no-answers before the first call that is answered. Then X will have a Geometric distribution with $p = 0.4$. To try three calls before having a call answered means that we will have three failures followed by one success:

$$\mathbf{P}(X = 3) \;=\; (1 - 0.4)^3(0.4) \;=\; (0.6)^3(0.4) \;=\; 0.0864$$

(b) At most three calls before getting the first call answered?

$$\begin{aligned}
\mathbf{P}(X \leq 3) &= \mathbf{P}(X = 0) + \mathbf{P}(X = 1) + \mathbf{P}(X = 2) + \mathbf{P}(X = 3) \\
&= (1 - 0.4)^0(0.4) + (1 - 0.4)^1(0.4) + (1 - 0.4)^2(0.4) + (1 - 0.4)^3(0.4) \\
&= (0.4)\big(1 + (1 - 0.4) + (1 - 0.4)^2 + (1 - 0.4)^3\big) \\
&= 0.4\big(1 + 0.6 + 0.6^2 + 0.6^3\big) \\
&= 0.8704
\end{aligned}$$