Tutorial 1

September 24, 2020

Question 1

Suppose we draw 2 cards (without replacement) from a deck of 52 cards.

Let A = "the first card is an ace"
Let B = "the second card is a spade"

Are A and B independent events?

Question 2

Two players, **A** and **B**, are shooting at a target simultaneously and independently. For each round, the probabilities of hitting the target are:

- 1/2 for player **A**
- 1/4 for player **B**

The game continues until the target is hit (i.e. if both miss, another round is played). The game ends if, in a particular round, any of the following occur:

- **A** hits, **B** misses. **A** wins.
- **B** hits, **A** misses. **B** wins.
- **A** hits, **B** hits. **A** and **B** tie.

What is the probability of **A** winning the game?

Question 3

A student is taking an exam that has a one hour time limit. Suppose the probability that the student finishes the exam in less than x hours is $x/2$, for all $0 < x < 1$. Then, given that the student is still working after 0.75 hours, what is the conditional probability that the full hour is used?

Question 4

Suppose we have five boxes, each containing four balls:

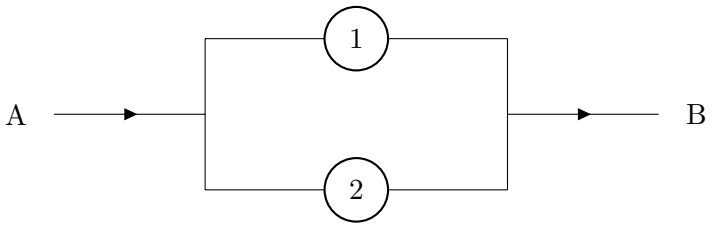
- Box 0: 4 white balls
- Box 1: 3 white balls, 1 black ball
- Box 2: 2 white balls, 2 black balls
- Box 3: 1 white ball, 3 black balls
- Box 4: 4 black balls

(a) A box is chosen at random and two balls are drawn **without replacement**. What is the probability that both balls are black?

(b) A box is chosen at random and two balls are drawn **with replacement**. What is the probability that both balls are black?

(c) A box is chosen at random and two balls are drawn **without replacement**. Given that both balls are black, what is the probability that they came from Box 2?

Question 5



(a) Suppose we have a system as above that functions if at least one of its components are functioning. It is known that:

- Component 1 functions 90% of the time
- Component 2 functions 80% of the time
- The states of the components are independent

What is the probability that the system is non-functioning?

(b) Now consider a system similar to (a) but with n parallel components. Once again, the system will function if at least one of its components are functioning, and the states of the components are independent.

Let C_i denote the event that component i is functioning, $i = 1, 2, \ldots, n$. What is the probability that the system is functioning?