Tutorial 2

October 1, 2020

Question 1

A random variable X takes on values 1, 2, 3, 4 such that

$$2\mathbf{P}(X=1) = 3\mathbf{P}(X=2) = \mathbf{P}(X=3) = 5\mathbf{P}(X=4)$$

Find the probability distribution function and cumulative distribution function of X.

Question 2

A contestant on a quiz show is presented with two questions, labelled 1 and 2, which he is to attempt in whichever order he chooses.

- If he attempts question i first, he will be allowed to proceed to question j ($j \neq i$) only if his answer to question i is correct.
- If his answer to i is incorrect, he is not allowed to answer question j.

The contestant is to receive V_i dollars for correctly answering question i, $i = 1, 2$. For example, the contestant will receive $V_1 + V_2$ dollars if he answers both questions correctly.

Let E_i, $i = 1, 2$ be the event that the contestant knows the answer to question i, and that the two events are independent. Let P_i, $i = 1, 2$ represent the respective probabilities of these events. In order for the contestant to maximize his expected winnings, which question should he attempt first?

Question 3

(a) A particles moves n steps on a number line. The particle starts at 0, and at each step moves one unit to the right or the left, with equal probabilities. Assume all steps are independent. Let Y be the particle's position after n steps. Find the PMF of Y.

(b) Using the results from (a), let D be the particle's distance from the origin after n steps. Assume that n is even. Find the PMF of D.

Question 4

(a) If X takes values from $\{0, 1, 2, \ldots, n\}$ ($n = \infty$ is OK), show that

$$\mathbf{E}(X) = \sum_{k=1}^{n} \mathbf{P}(X \geq k)$$

(b) Four dice are rolled. Let M be the minimum of the four numbers. Find $\mathbf{E}(M)$.