Tutorial 4

October 15, 2020

Question 1

Let U_1 and U_2 be two independent Uniform$[0, 1]$ random variables, and let $X = \min(U_1, U_2)$ be the minimum between them. Show that the density of X is

$$f_X(x) = 2 - 2x, \quad 0 \le x \le 1$$

[Hint: Start by computing $F_X(x) = \mathbf{P}(X \le x)$.]

Question 2

A die is rolled 24 times. Let S_{24} represent the sum of the 24 numbers rolled. Using the central limit theorem, approximate $\mathbf{P}(S_{24} \ge 100)$.

Question 3

A die is rolled k times. Let S_k represent the sum of the k numbers rolled. Using the central limit theorem, how large should k be so that $\mathbf{P}(S_k \ge 100) > 0.05$?

Question 4

Using MGFs, show that the sum of two independent Poisson random variables is a Poisson random variable. What is the parameter of the new random variable?

Question 5

Using the MGF of the exponential distribution, obtain all the moments of the exponential distribution.

Question 6

Using the MGF of the standard normal distribution, obtain all the moments of the standard normal distribution.

Question 7

Let Z be a standard normal random variable. Compute the following probabilities:

(a) $\mathbf{P}(0 \leq Z \leq 2.17)$

(b) $\mathbf{P}(0 \leq Z \leq 1)$

(c) $\mathbf{P}(-2.50 \leq Z \leq 0)$

(d) $\mathbf{P}(-2.50 \leq Z \leq 2.50)$

(e) $\mathbf{P}(Z \leq 1.37)$

(f) $\mathbf{P}(-1.75 \leq Z)$

(g) $\mathbf{P}(-1.50 \leq Z \leq 2.00)$

(h) $\mathbf{P}(1.37 \leq Z \leq 2.50)$

(i) $\mathbf{P}(1.50 \leq Z)$

(j) $\mathbf{P}(|Z| \leq 2.50)$

Question 8

Consider the random sum

$$X = X_1 + X_2 + X_3 + \ldots + X_N$$

where

$$X_i \overset{\text{iid}}{\sim} \text{Exp}(\lambda) \quad \text{and} \quad N \sim \text{Geometric}(p), \quad X_i \perp N$$

Determine the distribution of X by finding its MGF.