

Tutorial 6

November 5, 2020

Question 1

Determine the value of C such that:

$$f(x, y) = \begin{cases} C(x + y) & 0 < x < 3, \quad x < y < x + 2 \\ 0 & \text{otherwise} \end{cases}$$

is a valid joint PDF. Then compute the following:

- (a) $\mathbf{P}(X < 1, Y < 2)$
- (b) $\mathbf{P}(Y > 2)$
- (c) $\mathbf{E}(X)$

Question 2

Let X and Y be independent $N(0, 1)$ distributed random variables. Show that $X + Y$ and $X - Y$ are independent $N(0, 2)$ distributed random variables.

Question 3

The joint density of X and Y is given by:

$$f(x, y) = \begin{cases} e^{-(x+y)} & 0 < x < \infty, \quad 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find the density of the random variable X/Y .

Question 4

The joint PDF of X and Y is given by:

$$f(x, y) = \begin{cases} e^{-(x+y)} & x > 0, \quad y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the PDF of $U = \frac{X + Y}{2}$.