

# Tutorial 7

November 12, 2020

## Question 1

Let  $X$  and  $Y$  be independent  $N(0, 1)$  distributed random variables. Show that  $X + Y$  and  $X - Y$  are independent  $N(0, 2)$  distributed random variables.

## Question 2

The joint PDF of  $X$  and  $Y$  is given by:

$$f(x, y) = \begin{cases} e^{-(x+y)} & x > 0, \quad y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the PDF of  $U = \frac{X + Y}{2}$ .

## Question 3

Suppose that two random variables  $X_1$  and  $X_2$  have the following joint distribution:

$$f(x_1, x_2) = \begin{cases} 4x_1x_2 & 0 < x_1 < 1, \quad 0 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the joint pdf of the new random variables

$$Y_1 = \frac{X_1}{X_2} \quad Y_2 = X_1X_2$$

What is the marginal density of  $Y_1$ ?

## Question 4

Continuing from Question 3, find the marginal of

$$Z_1 = \frac{X_1}{X_2}$$

by first transforming to  $Z_1$  as above, and  $Z_2 = X_1$ , and then integrating  $z_2$  out of the joint pdf.