

Tutorial 8

November 19, 2020

Question 1

Consider an infinite series of Bernoulli(p) trials.

- (a) Find the covariance between X , the number of successes in trials 1 - 100, and Y , the number of successes in trials 51 - 150.
- (b) Find the covariance between X , the number of successes in trials 1 - 100, and Z , the number of failures in trials 51 - 150.
- (c) Find the correlation between Y and Z .

Question 2

X and Y are independent random variables with probability density functions:

$$f_X(x) = \begin{cases} 4ax & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad f_Y(y) = \begin{cases} 4by & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the correlation coefficient between $X + Y$ and $X - Y$.

Question 3

Let $U \sim \text{Unif}(-1, 1)$ and $V = 2|U| - 1$.

- (a) Find the distribution of V .
- (b) Show that U and V are uncorrelated but not independent. This example illustrates that knowing the marginal distributions of two random variables does not determine the joint distribution.

Question 4

Pick a point uniformly distributed in the triangle $x \geq 0, y \geq 0, x + y \leq 1$. Compute

- (a) $\mathbf{E}(X | Y = y)$
- (b) $\mathbf{E}(Y | X = x)$