

Tutorial 9

November 26, 2020

Question 1

You toss a fair coin repeatedly.

- (i) What is the expected number of tosses until the pattern HT appears for the first time?
- (ii) What is the expected number of tosses until the pattern HH appears for the first time?

Question 2

An immortal drunk man wanders around randomly on the integers. He starts at the origin, and at each step he moves 1 unit to the right or 1 unit to the left, with equal probabilities, independently of all his previous steps. Let b be a googolplex (that is, 10^g where $g = 10^{100}$ is a googol).

- (a) Find a simple expression for the probability that the immortal drunk visits b before returning to the origin for the first time.
- (b) Find the expected number of times that the immortal drunk visits b before returning to the origin for the first time.

Question 3

Let $\mathbf{X} = (X_1, X_2, X_3)$ have a multivariate normal distribution with mean vector $\mathbf{0}$ and variance-covariance matrix

$$\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Find $\mathbf{P}(X_1 > X_2 + X_3 + 2)$.

Question 4

Let X, Y, Z be i.i.d. $N(0, 1)$. Find the joint MGF of

$$(X + 2Y, 3X + 4Z, 5Y + 6Z)$$